quæ erunt Æquationis datæ Radices omnes reales; hæ nempe ad dextram erunt Radices affirmativæ, illæ verò ad ſiniſtram Radices negativæ. Demonſtratio eſt manifeſta ex præcedentibus, habita tantùm ratione Parabolæ per puncta B, C, c, κ, κ tranſeuntis. Nam poſito F foco Parobolæ, (cujus diſtantia à Vertice aſt $\frac{1}{4}$ ON,) notum eſt quod lineæ omnes ut FB + BQ, FC + CD, &c, eandem ubique conficiant ſummam.

Atque ex principiis hic poſitis proclive erit Inſtrumentum haud inconcinnum & quantumvis accuratum fabricari, cujus beneficio hujuſmodi Æquationum quarumcunque Radices nullo fere negotio inveniri poſſint, & præ oculis exhiberi. Hoc autem quilibet, ſi id Curæ ſit, variis modis pro ingenio ſuo efficere poteſt, & de his jam ſatis.

III. *Æquationum quarundam Poteſtatis tertiæ, quintæ, ſeptimæ, nonæ, & ſuperiorum, ad infinitum uſque pergendo, in terminis finitis, ad inſtar Regularum pro Cubicis quæ vocantur* Cardani, *Reſolutio Analytica.*

Per Ab. De Moivre, R. S. S.

SIt n Numerus quicunque, y quantitas incognita, ſive Æquationis Radix quæſita, ſitque a quantitas quævis omnino cognita, ſive ut vocant Homogeneum Comparationis: Atque horum inter ſe relatio exprimatur per Æquationem

$$ny + \frac{nn - 1}{2 \times 3} ny^3 + \frac{nn - 1}{2 \times 3} \times \frac{nn - 9}{4 \times 5} ny^5 + \frac{nn - 1}{2 \times 3} \times \frac{nn - 9}{4 \times 5} \times \frac{nn - 25}{6 \times 7} ny^7, \&c. = a$$

Ex

Ex hujus ſeriei natura manifeſtum eſt, quod ſi n ſumatur numerus aliquis impar (integer ſcilicet, nec refert utrum ſit affirmativus vel negativus) tunc ſeries ſponte ſua terminabitur, & Æquatio fit una ex ſupra præfinitis, cujus Radix eſt

$$(1)\quad y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}+a}}$$

$$\text{vel } (2)\quad y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{vel } (3)\quad y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{vel } (4)\quad y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}$$

Exempli gratia, ſit hujus Æquationis poteſtatis quintæ $5y + 20y^3 + 16y^5 = 4$ Radix invenienda, quo in caſu erit $n = 5$ & $a = 4$. Radix juxta formam primam erit $y = \frac{1}{2}\sqrt[5]{\sqrt{17}+4} - \frac{\frac{1}{2}}{\sqrt[5]{\sqrt{17}+4}}$, quæ in numeris vulgaribus expeditiſſime explicari poteſt ad hunc modum. Eſt $\sqrt{17} + 4 = 8.1231$, cujus Logarithmus 0.9097164, & hujus pars quinta 0.1819433, huic reſpondens numerus eſt $1.5203 = \sqrt[5]{\sqrt{17}+4}$. Ipſius vero 0.1819433 Complementum Arithmeticum eſt 9.8180567. cui reſpondet numerus $0.6577 = \frac{1}{\sqrt[5]{\sqrt{17}+4}}$ Igitur horum numerorum ſemidifferentia $0.4313 = y$.

Hic venit Obſervandum quod loco Radicis generalis, non incommode ſumeretur $y = \frac{1}{2}\sqrt[n]{2a} - \frac{\frac{1}{2}}{\sqrt[n]{2a}}$, ſi quando numerus a reſpectu unitatis, ſi ſatis magnus, ut ſi Æquatio fuerit $5y + 20y^3 + 16y^5 = 682$, erit Log. $2a = 3.1348143$, cujus pars quinta 0.6269628, & huic reſpondens numerus 4.236. Complementi autem Arithmetici 9.3730372 numerus eſt 0.236 & horum numerorum ſemidifferentia $2 = y$.

Atqui præterea, ſi in Æquatione præcedenti ſigna alternatim ſint affirmantia & negantia, vel quod eodem redit, ſi ſeries obvenerit hujus modi

$$ny + \frac{1-nn}{2\times3}ny^3 + \frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}ny^5 + \frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}\times\frac{25-nn}{6\times7}ny^7, \&c. = a$$

erit hujus Radix

$$(1)\qquad y = \frac{1}{2}\sqrt[n]{a+\sqrt{aa-1}} + \frac{\frac{1}{2}}{\sqrt[n]{a+\sqrt{aa-1}}}$$

$$\text{vel }(2)\qquad y = \frac{1}{2}\sqrt[n]{a+\sqrt{aa-1}} + \frac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}$$

$$\text{vel }(3)\qquad y = \frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}} + \frac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}$$

$$\text{vel }(4)\qquad y = \frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}} + \frac{\frac{1}{2}}{\sqrt[n]{a+\sqrt{aa-1}}}$$

Hic autem Notandum, quod ſi $\frac{n-1}{2}$ numerus extiterit impar, Radicis inventæ ſignum in ei contrarium permutandum eſt.

Proponatur Æquatio $5y - 20y^3 + 16y^5 = 6$, unde $n = 5$ & $a = 6$. Erit Radix $= \frac{1}{2}\sqrt[5]{6 + \sqrt{35}} + \frac{\frac{1}{2}}{\sqrt[5]{6 + \sqrt{35}}}$

Vel quoniam $6 + \sqrt{35} = 11.916$, erit hujus logarithmus 1.0761304 & ejus pars quinta 0.2152561, Complementum vero Arithmeticum 9.7847439. Horum Logarithmorum numeri ſunt 1.6415 & 0.6091 reſpective, quorum ſemiſumma $1.1253 = y$.

Verum ſi acciderit ut a ſit minor unitate, tunc Radicis forma ſecunda, ut quæ propoſito eſt magis conveniens, præ reliquis ſeligenda eſt. Sic ſi Æquatio fuerit $5y - 20y^3 + 16y^5 = \frac{61}{64}$, erit $y = \frac{1}{2}\sqrt[5]{\frac{61}{64} + \sqrt{\frac{-375}{4096}}} + \frac{1}{2}\sqrt[5]{\frac{61}{64} - \sqrt{\frac{-375}{4096}}}$. Et quidem ſi Binomialium Radix quintana ullo pacto extrahi queat, prodibit Radix proba & poſſibilis, etſi expreſſio ipſa impoſſibilitatem mentiatur. Binomialis vero $\frac{61}{64} + \sqrt{\frac{-375}{4096}}$ Radix quintana eſt $\frac{1}{4} + \frac{1}{4}\sqrt{-15}$, & Binomialis $\frac{61}{64} - \sqrt{\frac{-375}{4096}}$ Radix itidem quintana eſt $\frac{1}{4} - \frac{1}{4}\sqrt{-15}$, quorum Binomialium ſemiſumma $= \frac{1}{4} = y$.

Si autem extractio iſta vel non peragi poſſet, vel etiam difficilior videretur, res ubique confici poteſt per Tabulam ſinuum naturalium ad modum ſequentem.

Ad Radium 1 ſit $a = \frac{61}{64} = 0.95112$ ſinus arcus cujuſdam, qui proinde erit 72°: 23′ cujus pars quinta (eo quod $n = 5$) eſt 14°: 28′; hujus ſinus $0.24981 = \frac{1}{4}$ proxime. Nec ſecus procedendum in Æquationibus graduum ſuperiorum.